[This question paper contains 4 printed pages.]

(14) Your Roll No. 2023

Sr. No. of Question Paper : 4997

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : **B.A. (Prog.)**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions.
3. Attempt any **two** parts from each question.
4. Marks are indicated against each question.

1. (a) Form an equation whose roots are $-1, 2, 3 \pm 2i$. (6)

   (b) Solve the equation (6)

   $$x^3 - 13x^2 + 15x + 189 = 0,$$

   being given that one of the roots exceeds another by 2.

   (c) If α, β, γ, be the roots of the equation (6)

   $x^3 + 5x^2 - 6x + 3 = 0$, find the value of

P.T.O.

(i) $\sum \alpha^3$ (ii) $\sum (\alpha-\beta)^2$.

2. (a) Prove that : (6.5)

$2^{10}\cos^6\theta\sin^5\theta = \sin 11\theta + \sin 9\theta - 5\sin 7\theta - 5\sin 5\theta + 10\sin 3\theta + 10\sin\theta.$

(b) Sum the series : (6.5)

$\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \cdots$ to n terms, provided $\beta \neq 2k\pi$.

(c) State DeMoivre's theorem for rational indices and use it to solve the equation : (6.5)

$$x^7 - x^4 + x^3 - 1 = 0.$$

3. (a) Find the characteristic roots of the matrix A where (6)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

(b) Solve the system of linear equations (6)

$$2x - 5y + 7z = 6$$

$$x - 3y + 4z = 3$$

$$3x - 8y + 11z = 11$$

(c) Using Cayley Hamilton's Theorem, find the value of A^3, where (6)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$

4. (a) Let X and Y be two subspace of a vector space V. (6.5)

(i) Prove that the intersection $X \cap Y$ is also subspace of V.

(ii) Showthat the union $X \cup Y$ need not be a subspace of V.

(b) Let V = F[a, b] be the set of all real valued functions defined on the interval [a, b]. For any f and g in V, c in R, we define

$$(f + g)(x) = f(x) + g(x),$$

$$(c.f)(x) = cf(x)$$

Prove that V is a vector space over R, where R denotes the set of real numbers. (6.5)

(c) Show that the vectors $v_1 = (1,1,1)$, $v_2 = (1,1,0)$, $v_3 = (1,0,0)$ form a spanning set of $R^3(R)$, where R denotes the set of real numbers. (6.5)

5. (a) Find the multiplicative inverse of the given elements (if it exists) if it does not exist, give the reason

(i) [12] in Z_{16} (ii) [38] in Z_{83} (6)

(b) Find the order of each of the following permutations

(i) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 3 & 2 & 1 \end{pmatrix}$

(ii) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$

(c) Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a, b $\in$ G. (6)

6. (a) Prove that the set S = {0, 2, 4, 6, 8} is an abelian group with respect to addition modulo 10. (6.5)

(b) Let G be the group of all 2×2 invertible matrices with real entries under the usual matrix multiplication. Show that subset S of G defined by $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, b = c \right\}$, does not form a subgroup of G. (6.5)

(c) Show that $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$ is a subring of R, where R is a set of real numbers & Q is set of rational numbers. (6.5)

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(15) Your Roll No..............

Sr. No. of Question Paper : 5016

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : **B.A. (Prog.)**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions.
3. Attempt any **two** parts from each question.
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1. (a) Form an equation whose roots are 1, –1, i, –i. (6)

(b) Solve the equation

$$x^3 - 5x^2 - 16x + 80 = 0,$$

being given that the sum of two of its roots is zero. (6)

(c) Form the cubic equation whose roots are the values of α, β, γ given by the relations

P.T.O.

$$\alpha + \beta + \gamma = 3$$

$$\alpha^2 + \beta^2 + \gamma^2 = 5$$

$$\alpha^3 + \beta^3 + \gamma^3 = 11.$$

Hence find the value of $\alpha^4 + \beta^4 + \gamma^4$. (6)

2. (a) Prove that : (6.5)

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}$$

(b) Sum the series : (6.5)

$\cos\theta \sin\theta + \cos^2\theta \sin 2\theta + \cdots + \cos^n\theta \sin n\theta$ where $\theta \neq k\pi$.

(c) State DeMoivre's theorem for rational indices and use it to solve the equation : (6.5)

$$z^7 + z = 0$$

3. (a) Verify that the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

satisfies its characteristic equation and hence obtain A^{-1}. (6)

(b) Solve the system of linear equations (6)

$$x - 3y + z = -1$$

$$2x + y - 4z = -1$$

$$6x - 7y + 8z = 7$$

(c) Reduce the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{bmatrix}$$

to its normal form and then find its rank. (6)

4. (a) Show that the vectors $v_1 = (1,1,2,4)$, $v_2 = (2,-1,-5,2)$, $v_3 = (1,-1,-4,0)$ and $v_4 = (2,1,1,6)$ are linearly independent in $R^4(R)$. (6.5)

(b) Let V be the vector space of all $n \times n$ square matrices over a field F. Show that the set S of all symmetric matrices over Fisa subspace of V. (6.5)

(c) Let V be the set of ordered pairs (a, b) of real numbers. Let us define

$$(a, b) + (c, d) = (a + b, c + d)$$

$$\text{and} \quad k(a, b) = (ka, 0)$$

Show that V is not a vector space over R, where R is the set of real numbers. (6.5)

5. (a) Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 4 & 10 & 5 & 7 & 8 & 2 & 6 & 9 & 1 \end{pmatrix}$

as a Product disjoint cycles, construct its associated diagram and find its order. (6)

(b) State Euler's theorem. Hence, show that $23^{12} \equiv 1 (\text{mod} 28)$. (6)

(c) Let G = R -- {−1}. Define * on G by a * b = a + b + ab. Show that <G,*> is a group. (6)

6. (a) Let G = $GL_2(R)$. Show that $T = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}, ad \neq 0 \right\}$

is a subgroup of G. (6.5)

(b) Prove that rigid motions of a square yield the group S_4. (6.5)

(c) The set of Gaussian integers Z[i] = [a + bi, a, b ∈ Z} is a subring of the ring of complex numbers C. (6.5)